Tanta University

Department: Computers and Control Engineering
Total Marks: ٧٠ Marks

Faculty of Engineering

Course Title: **Fundamentals of Stochastic Processes** أسس العمليات العشوائية Course Code: CCE٣١١٧ ٣rd year
Date: ١٥.١.٢٠١٣ (First term) Allowed time: ٣ hrs No. of Pages: (٢)

Answer the following four questions. You are allowed to use the accompanying two tables of standard normal curve ordinates and areas in your answers.

Question No. ١ (١٦ marks)

(a) Let S={a, b, c, d, e, f} with P(a)=١/١٦, P(b)=١/١٦, P(c)=١/٨, P(d)=٣/١٦, P(e)=١/٤ and P(f)=٥/١٦. Let A={a, c, e}, B={c, d, e, f} and C={b, c, f}. Find:

i) $P(A/B)$.
ii) $P(B/C)$.
iii) $P(C/A^C)$.
iv) $P(A^C/C)$.

(b) Let A, B, and C be events. **Find an expression, and exhibit the Venn diagram,** for the event that:

i) A and B, but not C occurs.
ii) Only A occurs.

(c) In a certain college, ٢٥% of the boys and ١٠% of the girls are studying mathematics. The girls constitute ٦٠% of the students. If a student is selected at random and is studying mathematics, **determine the probability that the student is a girl?**

Question No. ٢ (١٨ marks)

(a) Find the expectation, variance, and standard deviation of the random variable x with density function P(x) given as:

x	١	٣	٤	٥
P(x)	٠.٤	٠.١	٠.٢	٠.٣

(b) Prove that for any random variable x:

i) $E(ax + b) = a\,E(x) + b$
ii) $V(ax + b) = a^٢\,V(x)$
iii) $E(c) = c$
iv) $V(c) = ٠$

where a, b, and c are constants.

(c) If the density function f(x) is given by:

$$f(x) = \begin{cases} ١-x & ٠ \le x \le ١ \\ x-١ & ١ \le x \le ٢ \\ ٠ & \text{elsewhere} \end{cases}$$

find the distribution function F(x).

Question No. ٣

(١٨ marks)

(a) A coin, weighted with P(H)= ٣/٤ and P(T)= ١/٤, is tossed three times. Let x be a random variable denoting the longest string of heads that occurs. **Find the distribution, expectation, variance, and standard deviation of x.**

(b) Consider the following binomial probability distribution:

$$P(x) = \binom{٥}{x} (٠.٧)^{x} (٠.٣)^{٥-x} \qquad (x = ٠, ١, \ldots, ٥)$$

where x is a random variable.

i) How many trials (n) are in the experiment?
ii) What is the value of p, the probability of success?
iii) Graph P(x).
iv) Find the mean and standard deviation of x.

(c) Suppose ٢% of items made by a factory are defective. **Find the probability that there are ٣ defective items in a sample of ١٠٠ items.**

Question No. ٤

(١٨ marks)

(a) Let x be a random variable with a standard normal distribution Φ. **Find:**

i) $P(x \geq ١.١٣)$
ii) $P(٠ \leq x \leq ١.٢٤)$
iii) $P(٠.٦٥ \leq x \leq ١.٢٦)$
iv) $P(-٠.٧٣ \leq x \leq ٠)$

(b) Let x be a random variable with the standard normal distribution Φ. **Determine the value of t, standard units, if:**

i) $P(٠ \leq x \leq t) = ٠.٤٢٣٦$
ii) $P(x \leq t) = ٠.٧٩٦٧$
iii) $P(t \leq x \leq ٢) = ٠.١٠٠٠$

(c) A class has ١٢ boys and ٤ girls. If three students are selected at random one after the other from the class, **what is the probability that they are all boys?**

Best wishes

Tanta University
Faculty of Engineering
Computers and Control Dept.

Third year students
First Term 2009/2010
Fundamentals of Stochastic Processes

Midterm exam

Question1

(1) Let A and B be events. Find an expression and exhibit Venn-diagram for the event that:

(i) A but not B occurs i.e. only A occurs.
(ii) Either A or B, but not both occurs.
(iii) A or not B occurs
(iv) Neither A nor B occurs

(2) Let a die be weighted so that the probability of a number appearing when the die is tossed is proportional to the given number let:

A= {even no.} B = {prime no.} C = {odd no.}

(i) Find the probability of each sample point of the sample space
(ii) Find P(A) , P(B) and P(C)
(iii) Find the probability that
  (a) An even or prime number occurs
  (b) An odd prime number occurs
  (c) A but not B occurs

(3) Let A and B be events with $P(A) = 1/3$, $P(B) = 1/4$, and $P(A \cup B) = 1/2$

Find: (i) $P(A \mid B)$ (ii) $P(B \mid A)$
(iii) $P(A \cap B^C)$ (iv) $P(A \mid B^C)$

(4) If the density function f (x) is given by:

$$f(x) = \begin{cases} 1-x & 0 \le x \le 1 \\ x-1 & 1 \le x \le 2 \\ 0 & \text{elsewhere} \end{cases}$$

Find the distribution function, and sketch both the distribution and the density functions.

Question2

(1) The probability that a man will live 10 more years is 1/4, and the probability that his wife will live 10 more years is 1/3. Find the probability that:

(i) both will be a live 10 more years
(ii) at least one will live 10 more years
(iii) neither will be live 10 more years
(iv) only the wife will live 10 more years

(2) Let X be a continuous random variable with the distribution

2/25

$$f(x) = \begin{cases} kx & \text{if } 0 \le x \le 5 \\ 0 & \text{elsewhere} \end{cases}$$

(i) Evaluate k (ii) Ffind $P(1 \le x \le 3)$, $P(2 \le x \le 4)$, and $P(x \le 3)$

Good Luck **Prof.Dr.E.Sallam**

Answer all the following questions;

Question No. 1

(a) If A and B are independent events , prove that A^c and B are independent.

(b) Let A and B be events with $P(A) = 1/2$, $P(B) = 1/3$ and $P(A \cap B) = 1/4$.

Find :i-$P(A|B)$, ii-$P(B|A)$, iii-$P(A \cup B)$, iv- $P(A^c|B^c)$, v- $P(B^c|A^c)$

(c) If X be a continuous random variable with the probability

$$P(x) = x/2 \qquad 0 < x < 2 \quad \text{, and zero elsewhere}$$

Find the **cumulative** distribution function, mean, variance, and standard deviation of X.

(d)Given a and b are constants , find with prove i - $E(a) = ?$ ii – $Var(a + b) = ?$

Question No. 2

(a) Three light bulbs are chosen at random from 15 bulbs of which 5 are defective.
Find the probability that : i- exactly one is defective, ii- none is defective,
iii- at least one is defective   iv- at most one is defective.

(b) Let X be a continuous random variable with distribution

$$f(x) = x/6 + k \qquad \text{if } 0 \le x \le 3 \quad \text{and } f(x) \text{ equals zero elsewhere.}$$

Sketch the graph of f(x) and thus i- Evaluate k   ii- Find $P(1 \le X \le 2)$

(c) A pair of fair dice is tossed. Let X assigns to the sum of dices numbers.
Calculate the mean, variance and standard deviation of X.

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(d Let X be a random variable with the binomial distribution $b(k;n,p)$.
Prove that $E(X) = np$.

Best wishes   ***Dr. Eng. Alasyed Sallam***

شاالله ما جبت اصفار

The Fundamentals of Stochastic processes

Sheet no.5

1) Medical research has shown that a certain type of chemotherapy is successful 70% of the time when used to treat skin cancer .suppose five cancer patients are treated with this type of chemotherapy and let x equal the no. of successful cures out of the five .

x	0	1	2	3	4	5
P(x)	0.002	0.029	0.132	0.309	0.360	0.168

The probability distribution of x is given in the following table.

Find:

a) $\mu = E(x)$

b) $\sigma = \sqrt{E(x-\mu)^2}$

2) Find the expectation, variance .and the standard deviation of each of the following:

i)

x	2	3	11
P(x)	1/3	1/2	1/6

ii)

x	-5	-4	1	2
P(x)	1/4	1/8	1/2	1/8

iii)

x	1	3	4	5
P(x)	0.4	0.1	0.2	0.3

(b) A coin weighted so that $P(H) = 1/3$ and $P(T) = 2/3$ is tossed until a head or four tails occur. Find the expected number of tosses of the coin.

(c) Determine the expected number of boys in a family with 8 children, assuming the sex distribution to be equally probable. What is the probability that the expected number of boys does occur?

(d) Let X be a random variable with the binomial distribution $b(k;n,p)$. Prove that $E(X) = np$.

Question No. 4

(18 marks)

(a) Determine the expected number of boys in a family with 8 children, assuming the sex distribution to be equally probable. What is the probability that the expected number of boys does occur?

(b) Suppose the diameters of bolts manufactured by a company are normally distributed with mean 0.25 inches and standard deviation 0.02 inches. A bolt is considered defective if its diameter is ≤ 0.20 inches or ≥ 0.28 inches. Find the percentage of defective bolts manufactured by the company. 93.49%

(b) Suppose the heights of 1000 male students are normaly distributed with mean 175 centemeters and standard deviation 20 centemeters.

Find the number of students with heights:

i- less than or equal to 130 centemeters, 12

ii- between 150 and 160 centemeters. 121

iii- between 170 and 180 centemeters 197

iv- greater than or equal to 200 centemeters. 106

Best wishes

Dr. Eng. Alasyed Sallam

Tanta University

Department: Computers and Control Engineering

Faculty of Engineering

Total Marks: 70 Marks

Course Title: **Stochastic Processes** العمليات العشوائية ثالثة حاسبات

Course Code: CCE3117 3rd year

Date: 4.2.2010 (First term)

Allowed time: 3 hrs

No. of Pages: (2)

Answer all the following questions:

Question No. 1

(17 marks)

(a) If A and B are independent events , prove that A and B^c are independent.

(b) Let A and B be events with P(A) =1/3, P(B) = 1/2 and $P(A \cap B)$=1/4.

Find :i-P(A|B) , ii-P(B|A) , iii-$P(A \cup B)$, iv- $P(A^c|B^c)$, v- $P(B^c|A^c)$

(c) If X be a continuous random variable with the probability

$P(x) = x/4 \quad 0 < x < 4$, and zero elsewhere

Find the **cumulative** distribution function, mean, variance, and standard deviation of X

(d)Given a and b are constants , find with prove i - E(a) = ? ii – Var(aX + b) = ? $\boxed{a^2 Var(x)}$

where X is a continuous random variable.

Question No. 2

(17 marks)

(a) Three light bulbs are chosen at random from 20 bulbs of which 5 are defective. Find the probability that : i- exactly one is defective, ii- none is defective,

iii- at least one is defective iv- at most one is defective.

(b) Let X be a continuous random variable with distribution

$f(x) = x/4 + k$ if $0 \le x \le 4$ and f(x) equals zero elsewhere.

Sketch the graph of f(x) and thus i- Evaluate k ii- Find $P(1 \le X \le 2)$ (3/4 - k)

k = -1/4

(c) A pair of fair dice is tossed. Let X assigns to the sum of dices numbers. Calculate the mean, variance and standard deviation of X.

(d) Let X be a random variable with the binomial distribution b(k;n,p).

Prove that E(X) = np.

Question No. 3

(18 marks)

(a) A fair die is tossed. Let X denotes twice the number appearing, and let Y denote 1 or 4 according as an odd or an even number appears. Find the probability, expectation, variance and standard deviation of:

i- X ii- Y iii- X+Y iv- XY

iv)

$$p(x)=\begin{cases}\frac{2}{25}x & 0\le x\le 5\\ 0 & elsewhere\end{cases}$$

3) Prove for any random variable x

i) E (ax+b) = aE(x) + b

ii) V (ax + b) = a^2v(x)

iii) E (c) = c

iv) V (c) =0

4) The heart association claims that only 10% of adults over 30 can pass the physical fitness test. Suppose that four adults are randomly selected and each is given the fitness test.

a) Find the probability that ~~three~~ none of the four adults pass the test

b) Find the probability that three of the four adults pass the test

c) Let x represent the number of the four adults who pass the test

d) Drive a formula for p(x), the probability distribution of the binomial random variable x.

5) Refer to problem 4. Use the formula for a binomial random variable to find the probability distribution of x, where x is the number of adults who pass the fitness test, graph the distribution

x	0	1	2	3	4
P(x)	0.6561	0.2916	0.0486	0.0036	0.0001

6) Refer to problem 5 .Calculate the mean and the standard deviation.

7) Give a formula for p(x) for a binomial random variable with n=7 and p= 0.2

8) Consider the following binomial probability distribution

$$P(x)=\binom{5}{x}(0.7)^{x}(0.3)^{5-x},\ X=0,1,2,3,4,5$$

a) How many trials n are in the experiment?

b) What is the value of p .the probability of success?

c) Graph p(x)

d) Find the mean and the standard deviation of x.

9) Suppose X is a binomial random variable with $n = 3$ and $p = 0.3$

a) Calculate the value of p(x), x=0, 1, 2, 3, using the formula for a binomial probability distribution.

b) Find the mean and the standard deviation of x

10) If x is a binomial random variable. Calculate mean, variance and standard deviation for each of the following

a) $n = 80$, $p = 0.2$

b) $n = 70$, $p = 0.9$

c) $n = 1000$, $p = 0.04$